Homework 10
Due: Wednesday, April 3
Math 435, Fall 2024

Problem 1. Tapp 1.6

> Exercise 1.6. If all three component functions of a space curve $\boldsymbol{\gamma}$ are quadratic functions, prove that the image of $\boldsymbol{\gamma}$ is contained in a plane.

(Recall that a plane is a 2-dimensional affine subset of $\mathbb{R}^3$, i.e., a subset defined by a single (not necessarily homogeneous) linear equation $ax + by + cz = d$, or equivalently a set of the form $\{\mathbf{p} + s\vec{u} + t\vec{v} \mid s, t \in \mathbb{R}\}$ for some $\mathbf{p}, \vec{u}, \vec{v} \in \mathbb{R}^3$).

Problem 2. Tapp 1.8

> Exercise 1.8. Compute the arc length of $\boldsymbol{\gamma}(t) = \big(2t, 3t^2\big)$, $t \in [0, 1]$.

Problem 3. Tapp 1.9

> Exercise 1.9. Let $a, b > 0$. Find the maximum and minimum speed of the ellipse $\boldsymbol{\gamma}(t) = (a\cos t, b\sin t)$.

Problem 4. Tapp 1.16

> Exercise 1.16. Let $\boldsymbol{\gamma}$ be a **logarithmic spiral**, as defined in Exercise 1.3 on page 6. Prove that the angle between $\boldsymbol{\gamma}(t)$ and $\boldsymbol{\gamma}'(t)$ is a constant function of t.

Problem 5. Let $\vec{x} = (0, 2), \vec{y} = (3, 4) \in \mathbb{R}^2$. Write $\vec{x}$ as a sum of two vectors, one parallel to $\vec{y}$ and the other orthogonal to $\vec{y}$.

Problem 6. Tapp 1.18

> Exercise 1.18. Let $\mathbf{x}$=$(1, 2, 3) \in \mathbb{R}^3$ and let $\mathcal{V}$=span$\{(1, 0, 1), (1, 1, 0)\}$. Write $\mathbf{x}$ as a sum of two vectors, one in $\mathcal{V}$ and the other orthogonal to every member of $\mathcal{V}$.

Problem 7. Tapp 1.24

> Exercise 1.24. If $t_0 \in I$ is the time at which the curve $\boldsymbol{\gamma} : I \to \mathbb{R}^n$ is farthest from the origin, prove that $\boldsymbol{\gamma}(t_0)$ is orthogonal to $\boldsymbol{\gamma}'(t_0)$.

Here, assume that I is an *open* interval.

(Recall that the geometric meaning of $\gamma(t_0) \perp \gamma'(t_0)$ is that $\gamma'(t_0)$ is orthogonal to the vector connecting $\gamma(t_0)$ to the origin.)